Microwaves

Series 10, solutions

Problem 1

We use a one coupler reflectometer to measure the reflection coefficient of a single port device, and we obtain $|b/b_{cc}| = 0{,}4$. We know that the used coupler has a coupled level of 10dB and a directivity of 26dB. Find the unknown reflection coefficient ρ, the percentage of power reflected by the device and it standing wave ratio (VSWR). Give the error margins for all three entities. (As the coupling level is moderate, give the exact error margins, not the approximated ones)

Solution

The error produce by the finite directivity is given by
$\left|\underline{S}_{31}/\underline{S}_{21}\underline{S}_{32}\right|$.

A directivity of 26 dB corresponds to $\left|\underline{S}_{31}/\underline{S}_{32}\right| = 10^{-26/20} = 0{,}05$.
If the coupling level is of 10 dB and the coupler is lossless, we have moreover

$$\left|\underline{S}_{21}\right| = \sqrt{1-\left(10^{-10/20}\right)^2} = 0{,}949$$

thus the error on $|\underline{\rho}|$ is 0,05/0,949 = 0,0527. We have thus the three parameters:
$0{,}3473 \le |\underline{\rho}| \le 0{,}4527$ $0{,}1206 \le |\underline{\rho}|^2 \le 0{,}2049$ and $2{,}0554 \le \text{ROS} \le 2{,}654$.

Problem 2

We want to realize a hybrid coupler in microstrip technology, similar to the coupler depicted in figure 6.39 of the handouts but with a 10dB power ratio between the direct (attenuated) and the coupled ports. Determine the characteristic impedances of the lines of the coupler, knowing that the impedance of the ports is 50 Ω.

Solution.
In order to have a 10 dB ratio between the direct and the coupled ports we need

$$\frac{\alpha^2}{\beta^2} = 10$$

Moreover, we $\alpha^2 + \beta^2 = 1$, thus

$\beta = \sqrt{\dfrac{1}{11}} = 0{,}3015$ and $\alpha = \sqrt{\dfrac{10}{11}} = 0{,}95346$

We then obtain the characteristic impedance of the branches of the coupler:

$Z_1 = \alpha Z_c = 0{,}95346 \cdot 50 = 47{,}67\,\Omega$ and $Z_2 = -\dfrac{\alpha Z_c}{\beta} = \dfrac{47{,}67\,\Omega}{0{,}3015} = 158{,}2\,\Omega$

Remark: The impedance Z_2 is high to be made in microstrip technology. it would imply a very narrow line, and thus high losses

remark 2: If we consider a 10 dB coupler instead of a coupler having a 10 dB ratio between the two outputs, we would have :

$\beta = \sqrt{\dfrac{1}{10}} = 0{,}316$ and $\alpha = \sqrt{\dfrac{9}{10}} = 0{,}9487$ and the characteristic impedances become

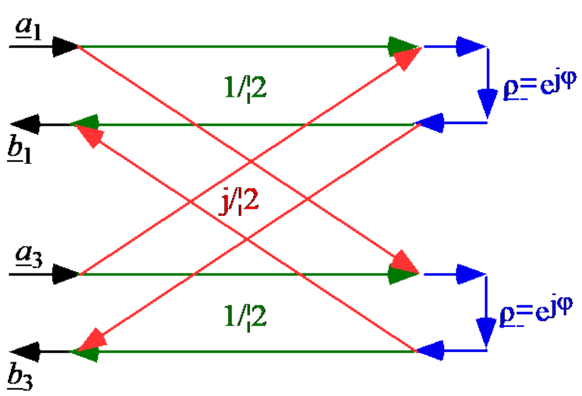
$Z_1 = \alpha Z_c = 0{,}99487 \cdot 50 = 47{,}435\,\Omega$ and $Z_2 = -\dfrac{\alpha Z_c}{\beta} = \dfrac{47{,}435\,\Omega}{0{,}316} = 150{,}1\,\Omega$

Problem 3

Find the scattering matrix of the device formed by a lossless symmetric coupler with α=β (hybrid coupler, scattering matrix given below), where ports 2 and 4 are terminated by identical total reflections. What is this device ?

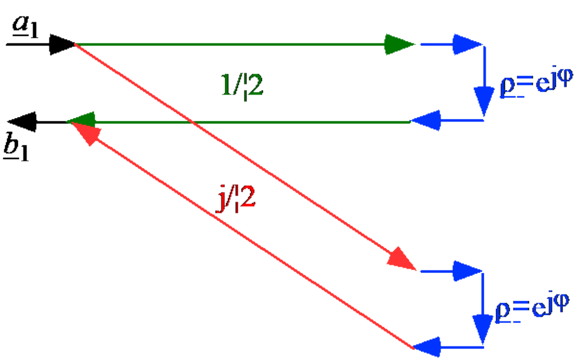
scattering matrix of the coupler : $[S] = \begin{bmatrix} 0 & \alpha & 0 & j\beta \\ \alpha & 0 & j\beta & 0 \\ 0 & j\beta & 0 & \alpha \\ j\beta & 0 & \alpha & 0 \end{bmatrix}$

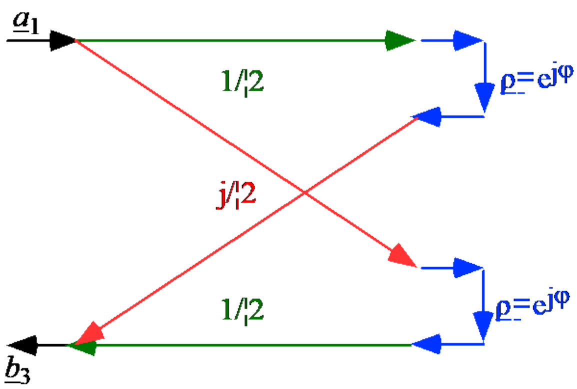
knowing that $\alpha^2 + \beta^2 = \alpha^2 + \alpha^2 = 1$ we find $\alpha = 1/\sqrt{2}$ and we draw the flow chart



For determining s_{11}, we consider all the possible paths between a_1 and b_1

$$\underline{b}_1 = \underline{a}_1 \cdot \frac{1}{\sqrt{2}} \cdot e^{j\varphi} \cdot \frac{1}{\sqrt{2}} + \underline{a}_1 \cdot \frac{j}{\sqrt{2}} \cdot e^{j\varphi} \cdot \frac{j}{\sqrt{2}} = \frac{\underline{a}_1 \cdot e^{j\varphi}}{2} - \frac{\underline{a}_1 \cdot e^{j\varphi}}{2} = 0$$





To determine s21, we consider all the possible paths between a_1 and b_2

$$\underline{b}_2 = \underline{a}_1 \cdot \frac{1}{\sqrt{2}} \cdot e^{j\varphi} \cdot \frac{j}{\sqrt{2}} + \underline{a}_1 \cdot \frac{j}{\sqrt{2}} \cdot e^{j\varphi} \cdot \frac{1}{\sqrt{2}} = \frac{j\underline{a}_1 \cdot e^{j\varphi}}{2} + \frac{j\underline{a}_1 \cdot e^{j\varphi}}{2} = j\underline{a}_1 \cdot e^{j\varphi}$$

The two other components are found through symmetry and reciprocity, yielding the scattering matrix

$$\begin{bmatrix} 0 & e^{j\varphi} \\ e^{j\varphi} & 0 \end{bmatrix}$$

The component is a matched lossless phase shifter